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Scaling of Optical and Low-Megahertz Acoustic Properties of Turbid-Water Systems in the Context of Image Quality

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DSTO-TN-0419

20030320 033

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
DOCUMENT CONTROL DATA

1. PRIVACY MARKING/CAVEAT (OF DOCUMENT)

2. TITLE

Scaling of Optical and Low-Megahertz Acoustic Properties of Turbid-Water Systems in the Context of Image Quality

3. SECURITY CLASSIFICATION (FOR UNCLASSIFIED REPORTS THAT ARE LIMITED RELEASE USE (L) NEXT TO DOCUMENT CLASSIFICATION)

Document	(U)
Title	(U)
Abstract	(U)

4. AUTHOR(S)

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6a. DSTO NUMBER	6b. AR NUMBER	6c. TYPE OF REPORT	7. DOCUMENT DATE
DSTO-TN-0419	AR-012-296	Technical Note	August 2002

8. FILE NUMBER	9. TASK NUMBER	10. TASK SPONSOR	11. NO. OF PAGES	12. NO. OF REFERENCES
490/6/120	NAV 01/041	DGMD	32	12

13. URL on the World Wide Web

http://www.dsto.defence.gov.au/corporate/reports/DSTO-TN-0419.pdf

14. RELEASE AUTHORITY

Chief, Maritime Operations Division

15. SECONDARY RELEASE STATEMENT OF THIS DOCUMENT

Approved for public release

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16. DELIBERATE ANNOUNCEMENT

No Limitations

17. CITATION IN OTHER DOCUMENTS   Yes

Acoustic imaging, Sonar arrays, Scaling laws, Scale models, Underwater photography

19. ABSTRACT

In the context of underwater imaging of minelike objects, it is desired to compare the quality of acoustic images with that of optical images at various turbidity levels. For this purpose, there has been interest in scaling experiments to a smaller size. First it is shown that, when modelling a target at range sl (e.g 5 metres), by experimenting at a shortened range l (e.g. 1 metre), one must (at l) use a different concentration of added matter for the acoustic than for the optical measurements. This effect arises because the acoustic attenuation that occurs in *clear* water is a constant and is not to be scaled. The report derives, first, the appropriate modified scaling laws, and second, the laws for maintaining constant image quality, on which the former laws depend. Because the image quality depends on the signal-to-noise ratio in the image, the latter laws depend on the properties of the acoustic noise. The laws of constant quality are derived for three noise environments: instrumentation noise, clutter and the combination of the two. Methods of carrying out the scaling experiment are described. Unfortunately the main conclusion is negative: the real noise is such that scaling experiments in the acoustic mine imaging context are not possible in practice. Here the problem for the first noise environment is that two assumptions are made that do not hold in the normal experimental arrangement; for the second and third environments, an acoustic array of reduced size would have to be built at prohibitive cost. Formulae for the visibility range are given.

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In the context of underwater imaging of minelike objects, it is desired to compare the quality of acoustic images with that of optical images at various turbidity levels. For this purpose, there has been interest in scaling experiments to a smaller size. First it is shown that, when modelling a target at range sl (e.g 5 metres), by experimenting at a shortened range l (e.g. 1 metre), one must (at l) use a different concentration of added matter for the acoustic than for the optical measurements. This effect arises because the acoustic attenuation that occurs in *clear* water is a constant and is not to be scaled. The report derives, first, the appropriate modified scaling laws, and second, the laws for maintaining constant image quality, on which the former laws depend. Because the image quality depends on the signal-to-noise ratio in the image, the latter laws depend on the properties of the acoustic noise. The laws of constant quality are derived for three noise environments: instrumentation noise, clutter and the combination of the two. Methods of carrying out the scaling experiment are described. Unfortunately the main conclusion is negative: the real noise is such that scaling experiments in the acoustic mine imaging context are not possible in practice. Here the problem for the first noise environment is that two assumptions are made that do not hold in the normal experimental arrangement; for the second and third environments, an acoustic array of reduced size would have to be built at prohibitive cost. Formulae for the visibility range are given.

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Executive Summary

In the context of underwater acoustic mine imaging (AMI), it is desired to compare the quality of the acoustic image with that of the corresponding optical image at various turbidity levels, in order to verify that acoustic imaging is indeed superior over much of the turbid region. For this purpose, it is not all that easy to secure the use of a tank for which two conditions are fulfilled: (i) the tank is large enough so that the experimental comparison can be carried out directly at the target ranges of interest; and (ii) the owner of the tank has no objection to suspended matter being added. Consequently there has been interest in scaling experiments to a smaller size. It has been hoped that from a small-scale experiment, inferences can be drawn about image qualities at larger ranges.

In this report, it is shown in steps that the scaling laws required are more complex than in normal scaling experiments. First it is shown that, when modelling the viewing of an object at range sl (e.g. 5 metres), by experimenting at a shortened range l (e.g 1 metre) one must (at l) use a *different* concentration of added matter for the acoustic than for the optical measurements. This effect arises because of the acoustic attenuation that occurs in *clear* water; this attenuation is a constant and is not to be scaled. Failure to use different concentrations results in a comparison that is unduly flattering to the AMI system.

Appropriate modified scaling laws are derived. In the course of this derivation, the laws for maintaining constant optical and constant acoustic image quality are also derived. Because the image quality depends on the signal-to-noise ratio in the image, the latter laws depend on the properties of the acoustic noise. The laws of constant quality are derived for three different noise environments, namely: instrumentation noise, clutter and the combination of the two. For each of these environments, one or more methods of carrying out the scaling experiment is described.

Unfortunately the main conclusion of the report is negative: *the real noise is such that scaling experiments in the acoustic mine imaging context are not possible in practice.* In the case of the first noise environment, the problem is that two assumptions are made that do not hold in the normal experimental arrangement. In the case of the second and third environments, an acoustic receiving array of reduced size would have to be built at prohibitive expense.

The options in regard to the optical-acoustic comparison therefore appear to be threefold. (i) The first option is to arrange for the use of a tank whose size comfortably exceeds 5 metres, so that no scaling is necessary. (ii) An alternative is to perform tests in a natural body of water. This is not an attractive option, because of the difficulty of controlling the sediment concentration. (iii) The final option is to postpone the

comparison until the AMI device has been built, then to make the comparison under operational conditions; this option entails accepting less than full control over the concentration.

Formulae for the visibility range are given.

Contents

1. Introduction

As a counter to the threat of sea mines in turbid water, the Acoustic Mine Imaging (AMI) program was initiated by the Maritime Operations Division (MOD) of DSTO [Jones 1996; Blair and Anstee 2000, and references therein]. The imaging system currently used is optical, and turbid conditions can severely affect its performance. By contrast, acoustic waves are much less affected by turbidity. Under the AMI program, an AMI sonar device is being developed by Thales Underwater Systems (TUS). This device, like the optical one, is to be mounted on a remotely operated vehicle (ROV).

As part of the AMI development work, the need has been identified to compare the quality of the acoustic images with the quality of the optical images at various turbidity levels and preferably also at various ranges.

The need for introducing scaling into such an experiment arises as follows. Operators count the optical image as degraded if the visibility range is less than 5 metres. Since 5 m marks a critical range, it would be desirable to do some of the experiments at this 5 m range. However, tanks as large as that are not readily available, especially if permission is to be gained to introduce suspended matter into the tank. *Hence there is interest in performing experiments at a range of around 1 metre and then, if possible, using a scaling law to infer what would happen if instead an experiment were performed at the larger range.* The aim of this report has been: (i) to determine whether a set of simple scaling laws applies and, (ii) if not, to determine whether more complex scaling laws apply and what the latter laws are. The achievement of the second aim has proved a much more complex task than expected.

Section 2 gives the optics background to such an experiment. In particular, it gives formulae for the visibility range in terms of the optical attenuation coefficient.

In this report, the 'scaled' system, or model system, means the (small) system upon which measurements are directly performed. The 'unscaled' system refers to the (large) system of real interest, for which indirect measurements are obtained by inference from the scaled system. Let s be the ratio of the range in the unscaled system to the range (after scaling) in the scaled system; s will be called the scaling factor for the range, or simply the scaling factor. In practice s is greater than one. Then Sections 3 and 4 obtain the formulae for the unscaled values of the parameters in terms of the scaled values; in other words the scaling laws are derived. However there is an important proviso to this claim: Sections 3 and 4 assume a certain 'law of constant image quality'; this law is later reviewed in Section 5.

Some conclusions of Sections 3 and 4 will now be given. An experiment involving scaling consists of two parts: a direct experiment (at a range of 1 m, say) on a model and an inference to the properties of an unscaled system (at, say, 5 m). We consider direct experiments in which a material of constant composition is added in varying concentrations to clear seawater. (Fresh water might be considered as a substitute.) Naively one might think that two assumptions hold. Assumption (i) is that 'simple scaling' prevails; that is, that all scaling is done by adjusting each parameter by a factor equal to one of s, unity or s^{-1}. Assumption (ii) is that, to draw conclusions about some state of the unscaled system, it suffices to perform a direct experiment using just *a single value of the concentration*, rather than making measurements with two concentration values. (A single-concentration experiment will be called a *'simple experiment.'*)

Both these assumptions turn out to be false. This happens because the acoustic attenuation coefficient consists of two components: a component independent of the concentration of added matter and a component proportional to that concentration.

In Section 3, the scaling laws for the optical properties are developed. Section 4 considers acoustic effects, particularly the acoustic attenuation γ. The section begins with a simple argument indicating that simple scaling cannot be correct due to the two-component nature of γ. Then, for a scaling factor s, the calculation of the unscaled values of the parameters, begun in Section 3, is completed. It is found that both simple scaling and the simple experiment must be abandoned as inadequate if a satisfactory scaling experiment is to be performed. In particular, to make inferences about an unscaled state, a different concentration must be used for the measurement of acoustic quality than for the measurement of optical quality.

So far it appears that indirect measurements of an 'unscaled' state *can* be made, the scaling laws however being more complex than one might have expected. But in fact the situation is much less rosy. In the work so far, an oversimple argument was used to derive, in both the optical and the acoustic case, 'the' law that relates images of constant quality. Section 5 attempts to give a more rigorous treatment of 'constant quality.' It is noted that for each situation (e.g. the optical situation), there is actually not a single law of constant quality but a set of such laws. In the optical case—i.e. a floodlit system—a detailed argument shows that indeed the originally-assumed constant-quality law survives as one of the set.

In the acoustic case, the set of constant-quality laws depends on the noise model assumed. For each of three noise models, a theoretical solution is found for the set of constant-quality laws. For each model, a theoretical solution is also found for the scaling laws of the experiment; they are little different from those deduced in Section 4. Unfortunately, for each model the theoretical solution does not represent a practical solution (or, in one case, represents a solution to an impractical noise model).

Section 6 describes how (at least in theory) the experiment, consisting of the optical and acoustic subexperiments, would be performed and how the results would be analysed. This is done for each of the three noise models. Also, three alternative versions of the experiment are described for one noise model only.

The conclusions are summarised in Section 7. The main conclusion is the negative one that scaling experiments with the AMI device are not possible in practice.

2. Relation of Visibility to Attenuation Coefficient

This section is concerned with optical imaging. The optical attenuation coefficient, denoted here by c, is defined [Williams 1970, p. 26; Jerlov 1976, p. 6] as the reciprocal of the distance in which the intensity[1] of a beam in a homogeneous medium drops by a factor $1/e$, where $e = 2.718$. The attenuation length is A.L. $= 1/c$.

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[1] This attenuation coefficient is thus twice what *could* have been defined as the attenuation coefficient, namely the reciprocal of the distance in which the *amplitude*, or rms electric field, of the beam drops by a factor $1/e$.

Williams [1970, pp. 78–80] defines the visibility range as the distance at which a given object can just be seen. He discusses the visibility range that is obtained under *natural lighting* (the sun). His derivation is based on the contrast in the image, in particular the smallest contrast that may be detected. As a result the *horizontal visibility range V* is given as

$$V = A/c \qquad (2.1)$$

where A varies from approximately 6 to about 2.5, depending on background luminance and object size. Williams continues, 'The usual rule of thumb is to pick some intermediate value; and very often one sees the visibility range given as

$$V = 3.5/c$$

or three and a half attenuation lengths, where 3.5 is apparently some average of the maximum and minimum values experienced in nature.' In other words, in the more general equation (2.1), the 'average' value of A is taken as

$$A = 3.5 \qquad (2.2)$$

This (average) value $A = 3.5$ is meant to apply to observations by the human eye. Apparently the value of A is a little less than this for a camera, which has less sensitivity.

Systems with one or more artificial light sources, or 'floodlights,' can be used when there is insufficient daylight (see Fig. 1 below). The light illuminates a cone, while there is another cone-shaped region to which the camera is sensitive. The limitation on such systems is the light scattered into the camera by the water. Such scattering is produced by scatterers in the common volume of the two cones. Williams (pp. 80–83) gives a qualitative discussion of floodlit systems. The video system used in mine imaging (the 'ROV video') normally involves a pair of floodlights together with the video camera (although daylight could be used in shallow water where there is sufficient sunlight).

Discussion within DSTO has led to the following tentative picture. For practical floodlit systems, the value of A varies from 1 to 3; it is thus less than the value 3.5 that applies to natural lighting. The value depends on the shape of the scattering function (versus angle) of the scatterers; the ratio of the backscattering coefficient to the total scattering coefficient being particularly relevant. The value of A depends also on the geometry of the video system (in particular, on the values of ε and y_0/l in Fig. 1). In the absence of measurements on the ROV video system, the best 'guesstimate' therefore appears to be

$$A = 2 \qquad (2.3)$$

Clearly however this figure is subject to considerable uncertainty.

Some miscellaneous comments will now be made. Williams (pp. 46–48) defines an additional quantity called the 'characteristic *extinction length*' and contrasts it with the attenuation length. The concept of extinction length has explanatory power; however, in Williams, it does not lead on to a general formula for visibility range that is superior to (2.1) combined with (2.2). We therefore put aside the concept of the extinction length for the purposes of this report.

The Secchi depth D (defined in the context of natural lighting), like V, is related to the attenuation coefficient. Hojerslev [1986] (quoted by Mulhearn [1993]) found that for monochromatic light

$$D = 6.3/c \qquad (2.4)$$

where the coefficient 6.3 must be some average, as in Equation (2.2).

Mulhearn goes on to make some comments relevant to the proposed scaling experiment. He takes 'turbid' waters to include *at least* those with visibility ranges of 6 m or less. He states: 'In turbid waters ... :

(i)   attenuation is dominated by scattering, and
(ii) c is independent of wavelength, to a good approximation in the visible region of the radiation spectrum [see, for example, Phillips and Scholz 1982, Fig. 5]'

(numbering like (i) added). The result (i) has been amplified by Kouzoubov [2001] and Kouzoubov [private communication] as follows. From measurements in Australian waters, the single-scattering albedo, or the ratio of the scattering coefficient to the total attenuation coefficient, is[2] $c_s/c \approx 0.7$.

We may add that, in turbid waters, the optical attenuation coefficient is proportional to the concentration of suspended matter and dissolved absorbents. In other words, if the concentration of all such attenuators is increased by a factor u, the attenuation coefficient c is also increased by a factor u.

As stated, the case $V = 5\,\text{m}$ is of particular interest for AMI. From Equations (2.1) and (2.3), for the floodlit system, the best estimates for c and the attenuation length are $0.4\,\text{m}^{-1}$ and 2.5 m respectively.

3. Scaling of Optical Properties

Consider the situation where the experimenter's real interest lies in a system that could in principle be constructed, but which, for practical reasons, is not constructed—to be called the 'unscaled' system. Instead, direct measurements are made on a related system, to be called the 'scaled' system; and inferences are made to the unscaled system. The fundamental quantity in the scaling is the range from the imaging device or receiver to the target or object being imaged. In the unscaled system the range l^+ is s times as large as the range l in the scaled or experimental system; thus

$$l^+ = sl \qquad (3.1)$$

(see row 1 of Table 1). The quantity s so defined will be called the scaling factor. To vary the optical attenuation coefficient c, a solid substance or mixture (e.g. kaolin) is added to the liquid in the experimental system; thus a suspension or solution (or a combination of the two) is produced. The superscript + will be used to denote an unscaled quantity.

In this section, while some acoustic properties are discussed, we concentrate on the scaling laws for the *optical* properties.

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[2] Thus Mulhearn's point (i) must be interpreted as saying that the scattering is greater than the absorption, not that the scattering is *much* greater than the absorption.

Table 1: *Showing the scaling of parameters when an inference regarding* optical *properties is made to a system (the 'unscaled' system) with a range s times that of the experimental system.*

Row	Parameter	Value in experimental system	Value in unscaled system
1	range	l	$l^+ = sl$
2	quality of optical image	q	$q^+ = q$
3	optical attenuation coefficient	c	$c^+ = c/s$
4	concentration of added matter	p	$p^+ = p/s$
5	transverse size of object being imaged	y	$y^+ = sy$

In this report we *define* 'clear' water as possessing two properties as follows. (i) 'Clear' water has an optical attenuation coefficient c of essentially zero. (ii) 'Clear' water has an acoustic attenuation coefficient γ equal to the inherent γ of seawater (not equal to zero), i.e. its value in the absence of bubbles, thermal microstructure, turbulence, etc., but particularly in the absence of suspended solids. This inherent value of γ is due overwhelmingly to absorption; as an indication, its value at 3 MHz and 20°C is $0.43\,\mathrm{m}^{-1}$ [Schulkin and Marsh 1963; Ishimaru 1978, pp. 56–57].

Property (i) requires further discussion. Adams *et al.* [1996] quote the value of c in the DSTO MOD Salisbury test tank as approximately $0.1\,\mathrm{m}^{-1}$. Both pure water and the clearest salt water are significantly less attenuating than this, since Adams *et al.* also quote $c = 0.05\,\mathrm{m}^{-1}$ (approximately) for the deep ocean. In the scaling experiment, the water used as the starting point (i.e. no 'added' matter) would probably have a value of c not far from the above test tank value. For a comparison, we need to know the typical value of c due to the added matter. Such a value can be derived from Section 6.4 below, which assumes Equation (2.3). Invoking also Equation (2.1) and taking $l = 1\,\mathrm{m}$ as a typical range in the experiment, we find that this c is $2\,\mathrm{m}^{-1}$. Comparing this 'typical value' with the value $0.1\,\mathrm{m}^{-1}$, we see that the relative error due to the neglect of the 'inherent' attenuation coefficient is only $1/20$, or 5%—*quite a small error.*[3, 4]

It is of interest to compare significant values of c and γ on the one scale. Thus we have the following four values.

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[3] If a more accurate treatment is required, it is expected that one could, without much difficulty, extend the theory of the present report by considering c to have two components, i.e. by writing $c = c_1 + c_2$, just as, at the beginning of Section 4, we write $\gamma = \gamma_1 + \gamma_2$.

[4] In future work, it may be necessary to review the finding that the 'inherent' value $0.1\mathrm{m}^{-1}$ can be disregarded on the grounds that the error in the attenuation coefficient is typically only 5%. The review may be necessary because, due to the exponential relationship, the resulting error in the *intensity of light* (and, therefore, in image quality) will be 20% over the typical go-and-return path of 2 m; thus, the error can be considerably increased over 5%.

$2\,m^{-1}$	estimated value of c at which, due to added matter, visibility range
	drops to 1 m (3.2)
$(0.43+\eta)m^{-1}$	value of γ at same concentration of added matter as in (3.2) above
$0.43\,m^{-1}$	inherent value of γ (acoustic)
$0.1\,m^{-1}$	test tank value ('inherent value') of c (optical)

In the second of the four values, η is a positive number that is believed to be small compared to 2. η depends on the composition of the suspended matter, but particularly on the typical diameter of the particles. It is known that, when the radius a of the particles is less than the wavelength λ, Rayleigh scattering applies and the scattered energy falls off rapidly with decreasing radius as a^6 [Morse and Ingard 1968, p. 427]. At higher a, the scattering is geometric and is proportional to a^2. η is small because, in natural waters, typically a/λ is much smaller for sound than it is for light (and often is not small at all in the case of light).

The above list shows that *in clear water, γ dominates c; but that in turbid water such that the visibility range is 1 metre, c (estimated to be $2\,m^{-1}$) dominates γ*. The water must be much more turbid still to bring γ up to the value $2\,m^{-1}$.

Besides the approximate visibility range formula (given by the combination of Eqns 2.1 and 2.3), the *second fundamental principle* connecting the unscaled system to the experimental system is as follows. The quality of the optical image is to be the same in the unscaled system as in the experimental system ($q^+ = q$, row 2). Here q is a number on some scale of optical image quality, set up by the experimenter. Essentially any scale that increases monotonically with quality serves the purpose.

The principle of equating the qualities leads to scaling laws for, first, the lateral size of objects, and second, the attenuation coefficient. We shall discuss these two quantities in that order.

In the scaling from one image (of an object at range l) to another (of an object at sl), we shall assume that *angular* sizes must be preserved. Thus an image of an experimental object of transverse size y is inverse-scaled to an image of an object of transverse size sy, as symbolised by row 5 of Table 1. In regard to the 'angular' assumption, it must first be remarked that it seems unnatural for the size of the object to be scaled in any other way. A more convincing argument can however be produced, based on the fact that the minimum detectable contrast between a coloured patch and its surroundings increases as the angular size of the patch gets smaller. If image quality is to depend only on the intensity of light and not on the range, the assumption must be accepted.

The scaling of the attenuation coefficient c can be discussed at a simple level or at a more rigorous level. At this stage we keep to the simple level as follows. A crude argument is that the quality of an image degrades only due to the decreasing brightness that it presents, which is proportional to $\exp(-cl)$. Thus the requirement $q^+ = q$ is achieved by making:

$$\exp(-cl) = \exp(-c^+l^+), \quad \text{i.e.} \quad cl = c^+l^+ \tag{3.3}$$

Hence, using row 1, we get $c^+ = c/s$ as in row 3.

A supporting argument for the conclusion (3.3) is given in Section A.1 of Appendix A. That section considers a situation in which the illumination is due entirely to daylight and all the attenuation is due to absorption rather than scattering. The more-or-less rigorous argument there does lead to the scaling law (3.3). However the situation considered in the argument is *somewhat different* from that in the experimental situation; hence this 'supporting argument' is still somewhat crude.

For the time being we accept Equation (3.3) as the scaling law that ensures constant quality of the image; shortly we shall tentatively accept also the analogous scaling law for acoustic propagation. On this basis the theory of scaling will be developed further in Section 4. In Section 5 we shall revisit the question of these two scaling laws and attempt to arrive at the truth by means of rigorous argument. In the optical case, we shall find that the scaling law (3.3) is vindicated—provided that it is adopted in conjunction with a wider *set* of scaling laws.

The scaling of the concentration p of added matter will now be considered. It is proportional to the attenuation coefficient c.[5, 6] (Any units of concentration on a linear scale will do.) Thus we have $c^+/c = p^+/p$; so we get $p^+ = p/s$ (row 4). (Note in Table 1 that each symbol *without* the superscript +, for example c, has, as its *normal* meaning, the value in the *experimental system*. But sometimes these symbols will be used *generically*, to mean, for example, attenuation coefficient in general.)

We shall see in Section 4 that, while p, c and q, satisfying the above relations, refer to quantities in the system on which *optical* measurements are made, they do not in general refer to the system on which *acoustic* measurements are made.

'Simple scaling' was defined in the Introduction as holding when each quantity is scaled by a factor of s, unity or s^{-1}. Note that all the scaling discussed in Section 3 has been simple scaling.

4. Effect of Acoustic Attenuation

Let γ denote the (total) acoustic attenuation coefficient. In the acoustic case, the quality is degraded from the value in a perfect medium ($\gamma = 0$) by two sources: (i) an acoustic attenuation coefficient γ_1 that is present already in clear water (see definition of 'clear'

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[5] The results given in this report hold even if there are dissolved colorants as well as suspended matter in the water, provided that a certain condition holds. The condition is that, throughout the range of concentrations used, each substance in the added matter is either completely dissolved or almost completely suspended. (This is because of the nonlinearity in the optical behaviour as the solution/suspension passes through the saturated state.)

[6] Despite the first sentence of the previous footnote, it is recommended that, if possible, a material that does not produce dissolved colorants be used. This is because otherwise there would be a bias introduced against optical devices (and in favour of acoustic devices) in some cases.

water in Section 3); and (ii) an acoustic attenuation coefficient γ_2 due to, and proportional to, the concentration of added matter.[7] We have

$$\gamma = \gamma_1 + \gamma_2 \qquad (4.1)$$

4.1 A Caution

Before the acoustic aspects of the scaling are considered in detail, a word of caution is in order. One must be wary of the procedure of naively inverse-scaling by a factor s, as was appropriate for the optical image. Consider the case where $s = 10$ and the scaled acoustic system has *clear* water (no suspension): specifically consider the situation in which the experimental range is $l = 1$ m and the unscaled range is 10 m. The acoustic image at 1 m suffers only a moderate degree of degradation due to the acoustic attenuation coefficient γ of the medium, from the list at Equation (3.2). Simple inverse-scaling by analogy with the scaling of the concentration p in Section 3 would imply that the same fairly good image quality is obtained at 10 m provided that the concentration satisfies

$$p^+ = p/s = p/10$$

which is zero, since the water in the scaled system is clear. But we know that in fact the conclusion that the image is fairly good is *false*, and that rather, the image degrades dramatically at 10 m. This degradation occurs because γ has two parts as follows. (i) γ_2 behaves normally, being proportional to the concentration (γ_2 happens to be zero in the present case). (ii) γ_1 has the property that γ_1 remains constant during the scaling, and so *the product $\gamma_1 l$ has increased by a factor of 10*. Therefore the acoustic intensity at 10 m contains a factor[8] $\exp(-\gamma_1 2 l^+)$, where, from the list at Equation (3.2), $2\gamma_1 l^+ = 8.6$. Thus the factor is equivalent to -37 dB and causes a large degradation in the image. Therefore the procedure of naively scaling as in Section 3 does not apply to the acoustic aspects of the experiment. It will soon be found that neither 'simple scaling' nor the 'simple experiment' applies when the experiment has acoustic aspects.

4.2 Scaling with Two Values of p

4.2.1 General

Let us see if we can devise a scaling experiment by making each experiment a double-barrelled one (i.e. a non-simple experiment). This is done by using one value of the concentration p of added matter for measurements of the optical image and a different value of p for the acoustic image. *Both subexperiments are to lead, by inference, to properties of the same unscaled, or enlarged, system,* in which the water is in one definite state. Let us call the concentrations in the optical and acoustic subexperiments p and p_a respectively.

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[7] Only the suspended matter, not the dissolved colorants, contributes to the *acoustic* attenuation. Proportionality to the concentration of total added matter holds, as well as proportionality to the concentration of suspended matter.

[8] The factor 2 is due to the two-way path.

In general, the parameter without a subscript or superscript will refer to the value in the experimental system in the *optical subexperiment*. A subscript a will be added for the *acoustic subexperiment* when the value for the parameter differs from that for the optical subexperiment. The third and fourth columns of Table 2 show how this naming is applied.

Table 2: Values of the parameters in the two subexperiments (third and fourth columns) and the one unscaled system (last column). The quantities are expressed in terms of the values in the optical subexperiment (together with the acoustic quantity r_a). Values of parameters in the unscaled system are represented by the insertion of the superscript +. Parentheses indicate that the quantity concerned has no physical significance for that particular subexperiment; this is true whether the value is given in the Table or not. The entries are based on the more accurate law for constant image quality, given by Equation (5.3). The predictions of the earlier assumption (4.3) are given by deleting the three terms containing $\ln s$.

Row	Parameter	Val. in optical subexp.	Value in acoustic subexperiment	Value in unscaled system
1	range	l	l	$l^+ = sl$
2	quality of optical image	q	$(\ )$	$q^+ = q$
3	optical attenu. coeff.	c	$(\ )$	$c^+ = c/s$
4	concen. of added matter	p	$p_a = p + \dfrac{(s-1)\gamma_1}{\delta} + \dfrac{\ln s}{\delta l}$	$p^+ = p/s$
5	transverse size of object	y	y	$y^+ = sy$
6	quality of acoustic image	$(\ )$	r_a	$r^+ = r_a$
7	acoustic attenu. coeff.	(γ)	$\gamma_a = s\gamma_1 + \gamma_2 + \dfrac{\ln s}{l}$	$\gamma^+ = \dfrac{s\gamma_1 + \gamma_2}{s}$
8	γ due to clear water	(γ_1)	γ_1	$\gamma_1^+ = \gamma_1$
9	γ due to added matter	(γ_2)	$\gamma_{2a} = (s-1)\gamma_1 + \gamma_2 + \dfrac{\ln s}{l}$	$\gamma_2^+ = \gamma_2/s$

The line of reasoning used to obtain a chain of values, culminating in the value of p_a, is captured in Table 2 (in which the three terms in $\ln s$ are to be ignored until a later section). The optical subexperiment is carried out to find (by inference) the quality of the optical image in the unscaled system; while the acoustic subexperiment is carried out to find the quality of the acoustic experiment in that same system. The last column of Table 2 describes this unscaled system; the values of the parameters in the unscaled system are denoted by the superscript +. The results in rows 1 to 5 (last column) are determined by optical considerations and are the same as in Table 1.

image. *For the AMI device to be useful, we require that, for the unscaled system, over a range of turbid conditions, $r^+ > q^+$ with a very clear margin.*

A way in which a common scale can be set up is as follows. The first step has been discussed by TUS and DSTO. A number of acoustic images (say 10) would be produced by taking a good image and degrading it digitally to produce distinct steps in quality. These 'standard' images would be given the 'r' values 1 to 10 (say). Then, given any other acoustic image, it would be assigned an r value according to the standard image that it most resembled in quality. (One could also interpolate between integers.) In the second step, one would simply rate the *optical* image on the 'scale of 1 to 10' set up for acoustic images, by deciding which of the standard acoustic images it is closest to in quality. (The fact that the optical image degrades in one way while the acoustic image degrades in a different way can lead to some subjectivity in the judgement; hopefully this subjective element is small.)

We now return to the filling-in of Table 2. Regarding row 6, we wish to devise the experiment so that the quality of the unscaled acoustic image is the same as in the acoustic subexperiment; hence r^+ in the fifth column must equal the value r_a in the fourth.

Next, the value of γ_1 is the same for all three systems, since it is simply the attenuation coefficient of clear water; this is shown in row 8. For this row, the value for the optical subexperiment has been placed in parentheses, because it has no physical significance for that subexperiment. The value is γ_1 nonetheless.

Note that Equation (4.1) can be extended as follows:

$$\gamma = \gamma_1 + \gamma_2, \quad \gamma_a = \gamma_{1a} + \gamma_{2a}, \quad \gamma^+ = \gamma_1^+ + \gamma_2^+$$

i.e.
$$\gamma = \gamma_1 + \gamma_2, \quad \gamma_a = \gamma_1 + \gamma_{2a}, \quad \gamma^+ = \gamma_1 + \gamma_2^+ \tag{4.2}$$

Now, since the value of the acoustic attenuation coefficient due to added matter for the *optical* subexperiment is defined as γ_2 (row 9), and the unscaled system has a concentration p/s, that attenuation coefficient becomes γ_2/s in the unscaled system (fifth column). The total coefficient γ^+ for the unscaled system (row 7) is then obtained by addition.

It remains to fill in three items[9] in the 'acoustic subexperiment' column. To do this we assume, by analogy with the optical case (Eqn 3.3), that the condition for constant quality of the acoustic image is

$$\gamma l = \text{constant}$$

i.e.
$$\gamma^+ l^+ = \gamma l \tag{4.3}$$

This condition will be reviewed in Section 5 from a more rigorous standpoint. First, it will be found that the scaling law (4.3) must be modified, but that the resulting modification to the law is manageable. Second, it will be found that the modified law is acceptable only if adopted as part of a *set* of scaling laws. Third, a question will be raised concerning the practicality of the experiment.

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[9] Constancy of angular size—the assertion that y_a, like y, is $s^{-1} y^+$ —has been taken over from the optical case. The detailed considerations of Section 5 give no reason to depart from this constancy.

As stated previously, the acoustic quality r_a is to remain unchanged from the fifth to the fourth column; therefore, from (4.3), the product γl remains unchanged. Since l scales by a factor $1/s$ (fifth to fourth column), γ must scale by a factor s; thus the value of γ_a in the fourth column, row 7, is determined. Now, in the acoustic subexperiment, $\gamma = \gamma_1 + \gamma_2$ must hold, so that row 9 is obtained for γ_{2a}:

$$\gamma_{2a} = (s-1)\gamma_1 + \gamma_2 \qquad (4.4)$$

The concentration p_a will be calculated in the next Section 4.2.2.

4.2.2 The Value of p_a

The values of the concentration p and the acoustic attenuation coefficient γ_2 due to that concentration is one of simple proportionality:

$$\gamma_2 = \delta p \qquad (4.5)$$

where the coefficient δ depends only on the chemical composition of the added matter. Note for future reference that we may similarly write for the corresponding optical property

$$c = dp \qquad (4.6)$$

where d is a constant.

Since γ_2/p must stay constant from the third column to the fourth, the value of p_a is obtained from row 9:

$$p_a = \frac{(s-1)\gamma_1 + \gamma_2}{\gamma_2}p \qquad (4.7)$$

Using (4.5), we can write this result in the more useful form

$$p_a = p + \frac{(s-1)\gamma_1}{\delta}; \qquad (4.8)$$

this is the result given in Table 2 (row 4).

Note that each of Equations (4.7) and (4.8) gives the *total* concentration for the acoustic subexperiment, not the concentration to be added to that which was present in the optical subexperiment. Note also from (4.8) that, provided $s > 1$, the concentration needed for the acoustic subexperiment always exceeds the optical-subexperiment concentration. Again, note that a *constant* extra concentration $(s-1)\gamma_1/\delta$ (independent of p) is to be added to the tank to change from the arrangement for the optical subexperiment to that for the acoustic subexperiment.

4.2.3 Discussion

Since the concentrations p and p_a in Table 2 are different, we have been forced to abandon the use of a *simple experiment*. The latter is defined as an experiment in which one obtains the properties of a single unscaled state by making the optical and the acoustic measurements at the *same* value of the concentration. We have also had to abandon *simple scaling*, since the scaling laws for p_a, γ_a and γ_{2a} (Table 2) do not simply involve a factor

scaling, since the scaling laws for p_a, γ_a and γ_{2a} (Table 2) do not simply involve a factor s, s^0 or s^{-1}. Note again that, although the two subexperiments are carried out with two different turbidities, after inverse scaling they give optical and acoustic results that refer to the same turbidity; indeed, to the concentration p/s (row 4 of Table 2).

4.3 Expression in Terms of Values in the Unscaled System

The laws for scaling with two values of p, given in Table 2, are better expressed by giving the formula for each quantity in terms of the values in the *unscaled* system (rather than the scaled optical system). The idea behind this 'new' arrangement is that one first decides the particular unscaled state for which determinations of values are to be made; then Table 3 shows what state must be used in the optical subexperiment and what state in the acoustic subexperiment. (In Table 3, again the three terms in $\ln s$ are to be ignored for the present.) The key result for the experimenter is row 4, giving the two different concentrations that must be used. Note that the arrangement is symmetric between the optical and the acoustic subexperiments.

Table 3: As for Table 2, except that all quantities are now expressed in terms of the values (such as l^+, γ^+) in the unscaled *system. The remarks concerning the terms in* $\ln s$ *continue to apply.*

Row	Parameter	Value in optical subexperiment	Value in acoustic subexperiment	Val. in unscal. system
1	range	$l = l^+/s$	$l = l^+/s$	l^+
2	quality of optical image	$q = q^+$	( )	q^+
3	optical attenu. coeff.	$c = sc^+$	( )	c^+
4	concen. of added matter	$p = sp^+$	$p_a = sp^+ + \dfrac{(s-1)\gamma_1^+}{\delta} + \dfrac{s \ln s}{\delta l^+}$	p^+
5	transverse size of object	$y = y^+/s$	$y = y^+/s$	y^+
6	quality of acoustic image	( )	$r_a = r^+$	r^+
7	acoustic attenu. coeff.	$(\gamma = \gamma_1^+ + s\gamma_2^+)$	$\gamma_a = s\gamma^+ + \dfrac{s \ln s}{l^+}$	γ^+
8	γ due to clear water	$(\gamma_1 = \gamma_1^+)$	$\gamma_1 = \gamma_1^+$	γ_1^+
9	γ due to added matter	$(\gamma_2 = s\gamma_2^+)$	$\gamma_{2a} = (s-1)\gamma_1^+ + s\gamma_2^+ + \dfrac{s \ln s}{l^+}$	γ_2^+

5. Variation of Image Quality Reconsidered

In this section, we use a more rigorous treatment to derive the laws governing images of constant quality. Such a derivation is given for one optical model and for each of three acoustic models.

5.1 Image Quality in a Floodlit System

In Section 3 it was assumed that, in the floodlit, optical system, the scaling law for constant image quality could be derived by a simple argument. That argument, based on the equality of the two attenuation factors, gave the scaling law (3.3). A 'back-up' argument, which was more rigorous but assumed conditions different from those in the actual experiment, was given in Section A.1.

To give a better treatment, the floodlit system is considered in some detail in Section A.2 of Appendix A. In this treatment, it is assumed that the light present from ambient sources is negligible, leaving only the illumination of the floodlights. The image quality is determined by the signal-to-noise ratio, where for this system the noise consists of *light scattered by scatterers in the water* (not the target) into the camera.

We choose the geometry of the optical imaging system (light source plus camera) to be *scaled by the same factor as the range*, in such a way that geometrical similarity holds (see Figure 1 below). Thus when an experiment is performed at a small range l, the system used is to be scaled down by a factor s from the ROV system that is of interest at $l^+ = sl$. Note that this scaling of the optical imaging system represents a *choice*; one could choose to keep the system of constant size. Whether the former is a good choice depends on its consequences; in particular, on whether the resulting scaling laws overall are simple. An experiment with the proposed scaling is not too difficult, since it turns out that the only quantity that needs to be altered in the scaling is the transverse *separation* between the floodlight and the camera. (The camera aperture itself, for example, need not be scaled as long as its linear size remains small compared with the separation.)

The argument in Section A.2 leads to the result that two scaling laws must be satisfied in order to ensure constant quality. The first is the proportionate scaling of the instrument, just described. The second law, perhaps surprisingly, is the same law (3.3) as originally 'derived' by a simple argument, namely

$$c^+ l^+ = cl \qquad (5.1)$$

These two laws are summarised in Table 4. (Strictly speaking, other 'obvious' scaling laws are also adopted, as discussed above Eqn A.9.) Actually, the situation is a little more complicated, as has been pointed out by a colleague.[10]

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[10] The modifications needed are as follows. In the context of scaling, the report mentions two parameters that need to be kept constant: the optical length cl and the geometrical configuration of the system (relative to the system's absolute size). But two further parameters must be kept constant to ensure proper scaling of the experiment: (i) the single-scattering albedo, that is, the ratio of the scattering coefficient to the total attenuation coefficient, and (ii) the scattering phase function that defines the angular distribution of the scattered light. The latter parameter defines the ratio of the backscattering coefficient to the scattering coefficient.

Table 4: *Showing the scaling laws that ensure constant quality of the floodlit, optical image. For ease of comparison, the layout is the same as in Table 5. In the present case, there are only two scaling laws. l is the range of the target.*

	Floodlit, optical
Sections	5.1, A.2
size of optical instrument	$y_0 \propto l$
c = optical attenuation coefficient	$cl = \text{const}$

5.2 Quality of Acoustic Image

In Section 4.2.1 (at Eqn 4.3), the 'law' for constancy of the *acoustic* image quality was obtained by the same over-simple argument as in the optical case. We now attempt a more rigorous discussion; the details are given in Section A.3.

The more rigorous investigation of the scaling laws in the acoustic case will be less complete than in the floodlit, optical case. There are two principal sources of noise: (i) *instrumentation (electronic) noise* and (ii) *clutter*, the latter being due to the distant sidelobes. Of these two, clutter has been found to be the dominant source of noise in the Project's AMI device, though significant instrumentation noise is present as well. Section 5.2.1 discusses the case where only instrumentation noise is present. The unfortunate situation revealed up to that point is reviewed in Section 5.2.2. Then Section 5.2.3 discusses the case where the only noise present is clutter. Finally Section 5.2.4 discusses the case where both sources of noise are present.

5.2.1 Instrumentation (Electronic) Noise

The case where the only noise that exists is instrumentation noise is discussed in detail in Section A.3.1. *It is assumed that this noise is independent of the transmitted power and of the received levels of voltage squared.* Then it turns out that one can use scaling in which *the receiving device is kept of constant linear size; thus the Project's AMI device can be used throughout.* When we impose also the requirement that the target is to be scaled in proportion to its range (preservation of angular size, as in Section 3), the scaling law for constant quality comes out to be

$$l^{+-2}\exp\left(-2\gamma^+ l^+\right) = l^{-2}\exp\left(-2\gamma l\right) \tag{5.2}$$

Taking the natural logarithm of (5.2), we obtain the following alternative forms:

$$\gamma^+ l^+ + \ln l^+ = \gamma\, l + \ln l \quad \text{or} \quad \gamma\, l + \ln l = \text{constant} \quad \text{or} \quad \gamma^+ l^+ = \gamma\, l - \ln s \tag{5.3}$$

where s is given by $s = l^+/l$. The law (5.3) replaces Equation (4.3).

Certainly, when the turbidity of the water is controlled by changing the concentration of the same additive component, these two parameters will remain approximately constant. However, consideration should be given to the match of these parameters between the experimental system and natural water.

The scaling method discussed so far is the preferred scaling method when the instrumentation noise dominates the clutter noise.[11] The method is summarised in Table 5.

Table 5: Showing the scaling laws that ensure constant quality of the acoustic image. Three situations are considered, as described by the headings; these situations are discussed in the sections listed in the next row. For each situation, the set of laws given is either the preferred set or the only set to ensure constancy of quality. The 'instrumentation noise' is subject to two assumptions, as explained in the text. In the 'instrumentation only' column, the first four laws ensure that the same acoustic instrument used in the unscaled system can be used in the scaled system. l is the range of the target.

	Instrumentation noise only	Clutter only	Combination of both noise sources
Sections	5.2.1, A.3.1	5.2.3, A.3.2	5.2.4, A.3.3
number of elements	$N = \text{const}$	$N = \text{const}$	$N = \text{const}$
area of an element	$\sigma = \text{const}$	$\sigma = \text{const}$	$\sigma = \text{const}$
level of instrumentation noise in the image	$n = \text{const}$	n irrelevant, since $n \approx 0$	$n = \text{const}$
size of acoustic array	$y_1 = \text{const}$	$y_1 \propto l$	$y_1 \propto l$
$\gamma =$ acoustic attenuation coefficient	$\gamma l + \ln l = \text{const}$	γ irrelevant	$\gamma l + \ln l = \text{const}$

Along the way in Section A.3.1, the option of scaling the receiving device proportionately to l is explored. (Such scaling has attractions, as Section 5.2.4 will show.) But it is found that there are two grave problems facing such a method of scaling. First, the idea of constructing a small-scale replica of the AMI device is quite impractical due to the expense and time involved—in fact such construction may well be technically impossible. Second, the instrumentation noise n must be made to vary with array size in a predictable way (or remain constant), and this implies a degree of control over the noise that is difficult to attain. For these two reasons, in practice an experiment based on proportionate scaling of the receiver size is impossible.

Let us return to the principal scaling method that is relevant when instrumentation noise is dominant, namely, the method based on a constant-size receiving device. It remains to examine the assumptions, that the noise is independent of the transmitted power and independent of the received power.

Discussions with TUS have made clear that these two assumptions do not hold in practice. However, it may be said that conceptually the two assumptions come close to holding. The latter result is so because the basic instrumentation noise is *electrical amplifier*

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[11] The method should also be suitable if it is the case that, while clutter is significant, one is able to, and one also wishes to, ignore the clutter and compare the degradation of the optical image with the degradation of the acoustic image *due to instrumentation noise alone.*

noise, which is indeed constant, that is, independent of the two powers. We may say that the two assumptions are correct, but with the two qualifications given below.

The first qualification causes the essential problem. This qualification is that, when *one-bit digitisation* is used — as it normally is in the AMI system — there is noise associated with that digitisation. This noise is not constant. Thus the analysis based on constant instrumentation noise is incorrect in practice. This noise requires further analysis. However, it is expected that one-bit digitisation puts an obstacle in the path of a valid scaling experiment in addition to the other obstacles identified in Section 5.

The second qualification is that an unplanned *'narrowband interference noise'* exists in the final device produced within AMI Phase 1. The dependence of this noise on quantities such as the transmitter power is unclear, but is a cause for concern. On the positive side, it is expected that this noise power will be removed in the AMI Phase 2 device.

5.2.2 Interlude

At this point we must admit a defeat in a certain sense. Even if we put aside the narrowband interference noise, the one-bit digitisation process has not been included in the analysis. In addition, the process of *correlation flattening*[12] is not included. If the latter two processes were included, it is conceivable that we might find a method of scaling that handles instrumentation noise; as it is, the handling of realistic instrumentation noise is not attempted. Actually, in the author's judgment, due to the nonlinear nature of these two processes, scaling that takes account of these processes is probably impossible. (From this point to the end of Section 6, the noise discussed will be taken not to include these two processes or narrowband interference.)

In the next Section 5.2.3, when we include clutter in the analysis, from a practical point of view we encounter a further defeat. As a result of these 'defeats,' the principal conclusions from this report will be *negative* ones: mainly, that scaling experiments with the AMI device are not possible in practice.

5.2.3 Clutter

We now consider the case where clutter is the dominant noise (see Section A.3.2 for details). The structure of the distant sidelobes may be described as a superposition of N ellipsoids (see Figure 2 below). The clutter is quite different from the other 'noises' considered in this report, because the attenuation factor $e^{-2\gamma l}$ affects the signal and noise equally.

In Section A.3.2 it is argued that constancy of *angular* size of the acoustic receiving device, as seen from the target, is a *necessary* condition for valid scaling. The section goes on to find the *totality* of conditions or laws needed for valid scaling in the clutter-only situation. These (theoretically possible) scaling laws for constant quality are believed to be given by Equation (A.17); they are repeated in Table 5. A feature of these scaling laws is that the attenuation constant γ does not enter.

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[12] In correlation flattening, a further one-bit digitisation is applied to each dechirped voltage stream immediately before beamforming. This reduces the clutter in the image.

Because proportionate scaling of the receiver is necessary, the first 'grave problem' in Section 5.2.1 is unfortunately applicable. Thus we can draw an important, though negative, conclusion: *in practice, scaling that takes account of the impact of clutter on quality is not possible.*

5.2.4 Combination of Instrumentation Noise and Clutter

The case where a combination of both sources exists is discussed in detail in Section A.3.3. It is argued that the scaling, given by Equations (A.15), (A.16) and (A.17), which is *theoretically* possible, can be applied. However that scaling, which requires constancy of the angular size of the array, is not possible in practice. That scaling is repeated in Table 5.

5.3 Resulting Changes in the Remaining Scaling Laws

Recall that Tables 2 and 3 give the relationships between the scaled and the unscaled quantities. Recall also that the relationships derived so far are obtained from Tables 2 and 3 by deleting the terms containing $\ln s$. We now derive the changes to Tables 2 and 3 as a result of the findings of this Section 5. (In fact the changes will be that the $\ln s$ terms are inserted.)

Consider first the situation dealt with in Section 5.2.1, in which the instrumentation noise is dominant and obeys a certain pair of assumptions. Then, from Table 5, the scaling laws for constant quality are that: (i) the entire array is held fixed, and (ii) Equation (5.3) holds. The law (i) simply means that, when we change from the unscaled to the scaled system or vice versa, the same (or an identical) array is to be used. Since the characteristics of the array do not enter into Tables 2 and 3, this requirement has no bearing on the completion of those tables. Because the new scaling law (5.3) has replaced (4.3), some of the remaining scaling laws, given in Tables 2 and 3, must be changed. We shall see that *the effect is to introduce the terms in* $\ln s$ *that have been ignored so far—three such terms in each table.*

First consider Table 2. By re-running the arguments of Sections 3 and 4, it is straightforwardly found that the correct table entries are the same as before, except for the three cells mentioned. To fill in these cells, we first note that, in the notation of Table 2, Equation (5.3) tells us that

$$\gamma^+ s l = \gamma_a l - \ln s \tag{5.4}$$

where we have used $l^+ = sl$. The use of the same arguments as in Section 4.2.1 yields

$$\gamma_2^+ = \frac{1}{s}[\gamma_{2a} - (s-1)\gamma_1] - \frac{\ln s}{sl} \tag{5.5}$$

The use of row 9 of the fifth column then yields the formula for γ_{2a} in Table 2 (row 9). Then the application of $\gamma_a = \gamma_1 + \gamma_{2a}$ immediately yields γ_a at row 7. (Alternatively, Eqn 5.4 yields the result directly.) The use of the relationship $p_a / p^+ = \gamma_{2a} / \gamma_2^+$ then yields the formula for p_a (row 4). This completes the amendments to Table 2.

From the completed Table 2, the completed Table 3 is straightforwardly obtained.

Now consider the case of combined noise (Section 5.2.4). Instead of the law (i) above (entire array held fixed), we have, in changing between the unscaled and the scaled array,

that the array size is scaled proportionately while N, σ and n are held constant. But, as argued in respect of (i), this scaling of the array has no bearing on Tables 2 and 3, which therefore remain the same as for instrumentation noise; the $\ln s$ terms are again included.

In the case where the only noise is clutter (Section 5.2.3), the consequences are peculiar. Here one is entitled to carry out the experiment as though the laws for combined noise were required to hold. Therefore Tables 2 and 3 again describe a valid experiment. The 'freer' scaling laws in the second-last column of Table 5 have essentially two consequences. First, when going to the scaled system, one would obtain the same results by allowing n to vary arbitrarily—provided that the instrumentation noise is still negligible compared to the clutter. Second, the concentration of added matter in the acoustic subexperiment could be varied arbitrarily, since the image quality would remain unaffected—subject only to the requirement that clutter remains the dominant noise. In the case of 'clutter only,' the experiment teaches us nothing about the effects of changing concentration on acoustic quality.

6. Plan of the Experiment

Four different versions of the experiment will be discussed in turn.

6.1 Version I

The experiment, in its complete form, consists of measurements as follows.
Preliminary measurements:

1. (optical) Measure the constant ratio $d = c/p$ introduced at Equation (4.6). Here a transmissometer[13] is used to measure c.

2. (acoustic) Measure the value of γ_1 —or accept the value 0.43 m^{-1} derived from the literature (Section 3).

3. (acoustic) Measure the constant ratio $\delta = \gamma_2/p$ introduced at Equation (4.5). This requires measuring one or more values of γ. The latter measurement might best be made by measuring the acoustic intensity due to a spherical transmitter at two ranges.

Main measurements:

4. The independent variable may be thought of as the parameter pair $\left(p^+, l^+\right)$ pertaining to the unscaled system. Select the values of this parameter pair to be used.

5. For each value of the pair $\left(p^+, l^+\right)$, proceed as below.

   a. Select the value of l to be used. (The scaled parameter l may be kept constant throughout the experiment without any information being foregone.)

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[13] A transmissometer of suitable length must be selected so that a suitably accurate measurement is obtained.

b. (optical)

   i. From the constancy of y_0/l in the scaling process (see the discussion preceding Eqn A.9), calculate the scaled value y_0 of the separation in the optical system; set up the experimental apparatus accordingly. The value y_0 of the separation is given by

$$y_0 = y_0^+/s = Y_0/s \qquad (6.1)$$

   where $s = l^+/l$ and Y_0 is the separation of light source and camera in the optical system of the actual ROV.

   ii. From p^+, l^+ and l, calculate the corresponding value of p in the optical subexperiment (Table 3). For that p, together with the selected l, measure[14] the image quality q.

c. (acoustic)

   i. Determine whether the scaling of Section 5.2.1, 5.2.3 or 5.2.4 is to be followed.[15]   In the first case, do not scale the acoustic instrument; in the latter two cases, scale the instrument according to Table 5.

   ii. From p^+, l^+ and l, calculate the corresponding value of p_a, the concentration in the acoustic subexperiment (Table 3). For that concentration, together with the selected l, measure the image quality r_a.

The *analysis* of the experimental results then proceeds as follows. The experiment gives q^+ and r^+ as a function of (p^+, l^+). Note that, in this unscaled system, the separation is fixed at Y_0 and the characteristic length of the acoustic instrument is fixed at Y_1. One could plot these outcomes as two sets on contours on the (p^+, l^+) plane; thus the two qualities can be compared. *For AMI to be useful, what is required is that r^+ exceeds q^+ by a considerable margin over a large part of the turbid region.* (In the nonturbid region it is expected, at this stage in the AMI development, that the inequality will be reversed, i.e. the optical system will be better.)

Note that a useful alternative may be to plot contours on the (p^+l^+, l^+) plane; for it may be that the quality depends mainly[16] on p^+l^+, so that the dependence on the second argument l^+ is weak. (Certainly for the optical case this is so, since the dependence is entirely on p^+l^+.)

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[14] As far as possible, the measurements in items 5b and 5c should, for convenience, be carried out in the order of increasing concentration.

[15] The reference to Section 5.2.1 is a reference to the preferred scaling within Sections 5.2.1 and A.3.1.

[16] This will be the case if the right-hand side of the second relationship in (6.5) has a variation with l^+ that is dominated by the p^+l^+ term.

As mentioned, the procedure for Version I can be carried out with l chosen to equal some fixed value l_2 throughout the experiment (say $l_2 = 1\,\text{m}$). However the analysis does not depend on the experimenter's choosing this option.

So far in Section 6, the discussion has encompassed both methods of scaling of the acoustic instrument size: $y_1 = \text{constant}$ and $y_1 \propto l$ (Table 5). The latter scaling is known to be impractical, so for simplicity we drop that option from the rest of Section 6. This forces us to restrict attention to the case where instrumentation noise is dominant and satisfies a certain pair of assumptions.

6.2 Version II

This version aims to simplify the experimental side of the procedure. There are two aspects of Version I that could be time-consuming, and Version II addresses both of these. First, in Version I the independent variable $\left(p^+, l^+\right)$ is two-dimensional; Version II reduces the independent variable to one dimension. Second, in Version I the separation y_0 must be varied as the experiment proceeds, so that many settings (or perhaps many optical instruments) are required; Version II keeps y_0 constant.

In Version II, first, l^+ *is fixed at some value* l_3 (say $l_3 = 5\,\text{m}$). Second, l is chosen to be fixed at some value l_2 (say $l_2 = 1\,\text{m}$). Thus s is fixed and consequently only one value of the separation $y_0 = Y_0/s$ is required.

The rest of the procedure is analogous to Version I. For each p^+, the values of p and p_a to be used in the two subexperiments are calculated from Table 3. The subexperiments determine q^+ and r^+. The outcomes may be plotted as two curves—graphs of q^+ and r^+ versus p^+. Finally r^+ is compared with q^+ as before.

As noted, the two advantages of Version II are: (i) there is one less dimension of independent quantities that must be varied, and (ii) only one setting of the video system is required. The *disadvantage* is that the results obtained are restricted to $l^+ = l_3$.

6.3 Version III

Just as Version II reduces the number of dimensions of data to be collected from two to one, Version III, at least apparently, also reduces the number of dimensions of data from two to one. But Version III has the further virtue of achieving this reduction without any loss of dimensionality in the *set of final results* obtained from the experiment. Unfortunately, it turns out that the first reduction—in dimensions of data—is apparent rather than real.

The argument is as follows. From (5.1), the quality of the optical image depends[17] *only* on the product cl, and thus only on pl. This result can be symbolised by

$$q \leftarrow pl \tag{6.2}$$

The symbols here are generic, as explained below. Similarly, from (5.2), the quality r of the acoustic image depends only on[18] $\gamma l + \ln l$; i.e.

$$r \leftarrow \gamma l + \ln l \tag{6.3}$$

To (6.3), we apply (4.2) and (4.5), finally divide the right-hand side by the constant δ to obtain[19]

$$r \leftarrow \frac{\gamma_1}{\delta} l + pl + \frac{\ln l}{\delta} \tag{6.4}$$

Note that the symbols in (6.2) to (6.4) are meant to be generic, and so we can immediately apply these relations to both the acoustic subsystem and the unscaled system to obtain, for example,

$$r_a \leftarrow \frac{\gamma_1}{\delta} l + p_a l + \frac{\ln l}{\delta}, \qquad r^+ \leftarrow \frac{\gamma_1}{\delta} l^+ + p^+ l^+ + \frac{\ln l^+}{\delta} \tag{6.5}$$

A simplification arises as follows. Each of q and r has been shown to be a function of a *one-dimensional* variable; that is,[20]

$$q = f_2(pl), \qquad r = h_2\left(\frac{\gamma_1}{\delta} l + pl + \frac{\ln l}{\delta}\right)$$

The determination of these two functional relationships, f_2 and h_2, is all that is required of the experiment; all the information that would be obtained from Version I can be deduced from f_2 and h_2. So finally, only an experiment with 'one dimension of input' is required; less experimenting is needed. So runs the argument.

But the argument has a flaw and the line of reasoning must be altered—though not discarded. The segment above (A.9) makes clear that the Equation (6.2) for the optical subexperiment holds only if both the value of y_0/l and the value of ε, characterising the optical device, remain constant. Presumably the experimenter will always make ε equal to the value on the ROV; but y_0/l remains to be controlled. To capture the requirement, then, (6.2) should be rewritten as

$$q \leftarrow pl, y_0/l \tag{6.6}$$

---

[17] Clearly the assertions of dependence and independence that follow are intended to hold for a given composition of the added matter.

[18] On the face of it, the expression is dimensionally improper. However, for any given system of units, the statements made are correct. Alternatively, we may replace $\ln l$ by $\ln[l/(1 \text{ metre})]$.

[19] From Equations (6.2) and (6.4) we see that two states with the same optical quality q^+ in general have two *different* acoustical qualities r^+.

[20] The use of f follows the convention of Section A.2 (see Eqn A.9), given that a 'hidden' dependence is about to be pointed out. h has been used for the other function (as at Eqn A.6) because there is no hidden dependence.

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Thus the optical subexperiment requires, after all, the input of a two-dimensional variable $(pl, y_0/l)$. Thus an experiment with two dimensions of input is required after all, so in this respect no gain on Version I has been made.

For the *acoustic* subexperiment, the dimensionality required for the inputs depends on which of the three acoustic scaling methods (Table 5) is used. First, if the method of either Section 5.2.3 or Section 5.2.4 is used, the input is two-dimensional, the second argument being y_1/l. But second, if the method described as 'preferred' within Section 5.2.1 (and Section A.3.1) is used—appropriate to the case where the instrumentation noise is dominant and obeys a certain pair of assumptions—the second input quantity is not required (it being implied that the same acoustic instrument is used throughout) and Equation (6.4) remains intact. In that case the experimental determinations needed are the functional relationships (6.6) and (6.4).

It follows that there is a situation in which Version III would be attractive, namely, when three conditions hold. The three conditions are: (i) that the preferred method within Section 5.2.1 is used, (ii) that y_0 is easy to vary experimentally, and (iii) that multiple measurements of optical quantities are easy to make compared to multiple measurements of acoustic quantities.

6.4 Version IV

Another idea for simplifying the experiment (starting from Version I) is to restrict attention to unscaled states for which the optical image is *just seen*. Then we may write

$$q^+ = q_b \tag{6.7}$$

where the subscript b stands for 'borderline.' Then once more the input variable in item 4 of Section 6.1 is reduced from two-dimensional to one-dimensional. Second, in addition to requiring (6.7), Version IV makes an assumption, by adopting the 'guesstimate' of the visibility range, given by Equations (2.1) and (2.3). Consequently no optical measurements need to be made!

This version has been proposed as a precise formulation of ideas that were put forward at a meeting involving TUS, DSTO and others.

The experiment proceeds by varying l^+ while holding to (6.7); l^+ then determines c^+ (from Eqns 2.1 and 2.3) and hence p^+. (Instead of varying l^+, one could vary p^+; but this procedure comes to the same thing, since p^+l^+ is held constant.) The scaled range l is chosen arbitrarily, and may as well be kept constant, equal to l_2, say. Then the scaled value p_a of the concentration is calculated; the acoustic subexperiment is performed to measure r_a, which is also r^+. Finally, r^+ (and the constant value of q^+) is plotted against l^+. r^+ is then compared with $q^+ (= q_b)$.

The advantages of Version IV over Version I are: (i) there is only one dimension of input quantities, and (ii) there is no need to perform an optical subexperiment. The disadvantages are as follows. (i) Data are obtained only for states in which $q^+ = q_b$. (ii) The procedure rests on the assumption $A = 2$ (Eqns 2.1 and 2.3). But, as in Section 2, A can vary considerably from 2. The value $A = 2$ (or any other assumed value of A) could

not safely be used for evaluating image quality. Thus Version IV as it stands is quite unsatisfactory.

6.5 Expected Results of the Experiment

Here we give some idea of what results might be expected from the experiment. For this purpose we may suppose that the measurements are made in a full-size tank, so that no scaling is required! We shall discuss only the case where the measured states are such that the optical image can just be seen (as in Version IV).

Consider various values of l^+ as follows (always with $q^+ = q_b$).

At $l^+ = 10$ m: From experience with the AMI, it is likely that the acoustic image at this range is worse than borderline, so we expect $r^+ < q_b$.

At $l^+ = 5$ m: A key requirement applies here. The performance of the AMI is to significantly exceed that of the optical system—at the turbidity for which the quality q^+ is borderline. Hence we want, and expect, r^+ to exceed q_b by a good margin.

At l^+ significantly less than 5 m: We want and expect the margin to be greater than at 5 m, with the margin presumably increasing as l^+ decreases. This is expected because $\gamma_1 l^+$ is decreasing, so the acoustic image is improving. (γ_2 is increasing, giving a contrary effect, but this effect is small, from Section 3.) The optical quality stays constant.

At $l^+ = 6, 7$ and 8 m: We hope that r^+ continues to exceed q_b up to as high an unscaled range l^+ as possible.

7. Conclusions

As discussed in Section 5.2, the principal conclusion of this report is negative: scaling experiments with the AMI device are not possible in practice. The main reason is that, once clutter is present as a source of noise, the size of the receiving array must be scaled in proportion to the range of the target—a scaling that is prohibitively expensive and perhaps technically impossible. In addition, one-bit digitisation and correlation flattening, though normally present in AMI experiments, have not been analysed. Each of the latter two processes[21] is likely to throw up an insuperable obstacle to satisfactory scaling.

In the course of the report, a number of problems have been solved. We have obtained a *theoretical* solution for each of three situations: first, the situation in which there is instrumentation noise only and that noise is subject to two assumptions (Section 5.2.1); second, where the only noise is clutter (Section 5.2.3); and third, where the combination of the preceding two noises exists (Section 5.2.4). For each situation, we have obtained the set of laws of constant image quality (Tables 4 and 5), and the set of scaling laws appropriate to the experiment as a whole (Tables 2 and 3). For each of the three situations, Section 6.1 describes how the scaling experiment, in its complete form, could, in theory, be performed

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[21] Narrowband interference noise is also in this category, at least temporarily.

and its results analysed. For the first of the three situations, Section 6 also describes three alternative versions of the scaling experiment.

8. Acknowledgements

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Appendix A: Variation of Image Quality

In this appendix the variation of image quality is studied, leading to the appropriate scaling laws at constant quality. To develop some of the concepts, in Section A.1 the scaling law is derived for a simple optical situation, which however differs from that in the experiment. The scaling laws relevant to the experiment are obtained in the optical, floodlit case in Section A.2. In the acoustic case, for each of three noise models, the relevant scaling laws are derived in Section A.3.

A.1. Image Quality in Daylight (with Absorption Only)

The argument given here is a modification of an argument given by Williams [1970, pp. 79–80]. When a diver views a scene *horizontally* in daylight, the contrast (in intensity) seen in an object at a range l is $C_0 \exp(-cl)$, where c is the attenuation coefficient and C_0 is a typical contrast[22] that the object would present in a medium with $c = 0$ (or at very small l). This exponential expression is the 'signal.' The quality q of the image should be determined by the signal-to-noise ratio, where the 'noise' is C_T, the smallest contrast that the eye may detect. The term 'noise' is appropriate here, because presumably there exists noise, of just this level, that prevents the eye from making a finer discrimination. C_T depends on both the background luminance and the angular size of the structure (in the object) that is to be detected. Thus, in the notation to be developed at Equation (6.2), we have

$$q \leftarrow (C_0/C_T)\exp(-cl) \tag{A.1}$$

Thus, for constant background luminance (daylight) and constant angular size (Section 3), two objects that are identically coloured (relative to their size), but in general placed in environments with different c and l values, present the same quality provided that

$$(C_0/C_T)\exp(-c^+l^+) = (C_0/C_T)\exp(-cl)$$

i.e. $$c^+l^+ = cl \tag{A.2}$$

Here one system has been taken as the unscaled system, its properties denoted by + superscripts.

The above argument assumes that there is no source of noise other than the 'minimum contrast' noise. Actually, in the situation of a diver viewing horizontally, there *is* another contribution to the noise if the scattering coefficient c_s —a contribution to c along with absorption—is nonzero. This is so because, then, daylight initially travelling vertically downwards (say) is scattered in all directions including into the camera. This factor will become of great importance in the case of a floodlit system, to be discussed in Section A.2. Meanwhile we can say that Equations (A.1) and (A.2) should both hold in the case where daylight conditions are coupled with the attenuation's being entirely due to absorption rather than scattering.

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[22] Whether the relevant contrast is internal to the object or external (i.e. the contrast is with the water) depends on circumstances. Clearly for an object of uniform colour the relevant contrast must be external. An internal contrast would arise, for example, from an object consisting of a checker-board pattern.

A.2. Image Quality in a Floodlit System

The floodlit system is simply the ROV optical system, but in general scaled in size. It is assumed that the light present from ambient sources is negligible, leaving only the illumination of the floodlights. The model assumed for the floodlit system is shown in Figure 1. The floodlight F emits light uniformly into a cone of half-apex angle ε. The camera C is uniformly sensitive to light from a cone, also characterised by ε. The separation of these two optical elements[23] is y_0. Both optical elements are assumed to have the axis of their cone pointed at right angles to the base plane FC. Note that, while here and below a number of assumptions are introduced, it is suspected that the argument can be modified to accommodate the dropping of many or all of these assumptions.
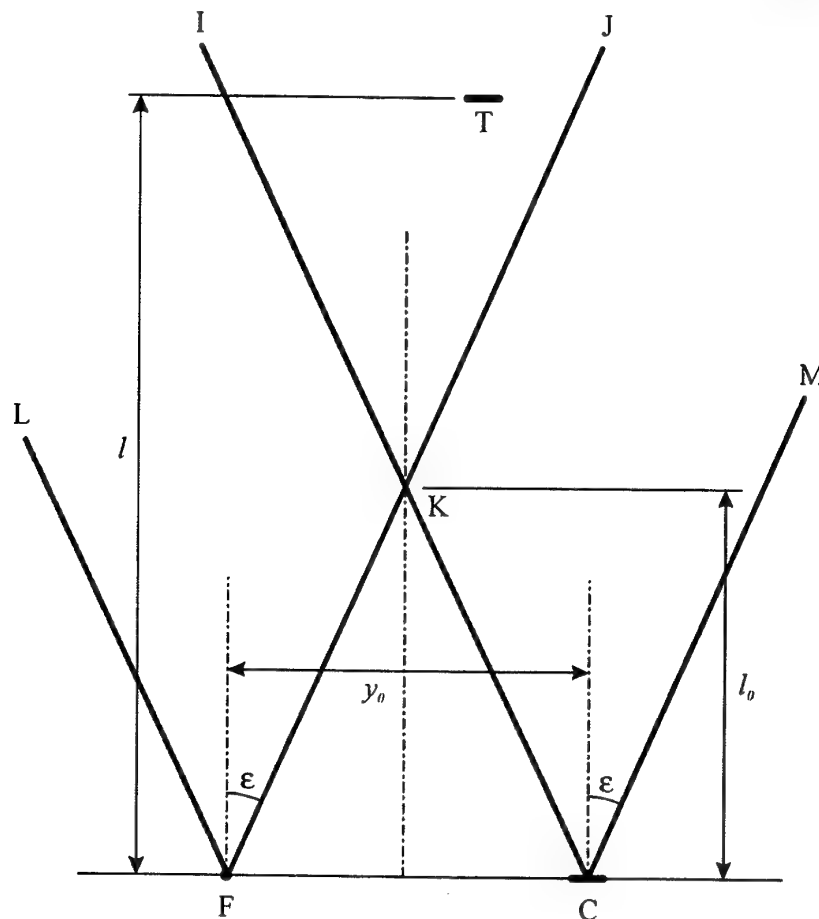

Figure 1: *The model of the floodlit system. The floodlight F emits light into a cone JFL. The volume to which the camera C is sensitive is the cone ICM. The target T must lie in the common volume IKJ of the two cones if it is to be seen.*

---

[23] In the actual ROV system there is a second floodlight, disposed so as to make the arrangement symmetric about the camera. It is believed that this second light makes no essential difference to the argument.

The target T must lie in the common volume of the two cones if it is to be seen. The distance from FC at which the common volume begins is denoted by l_0; note that

$$\tan \varepsilon = y_0/2l_0 \tag{A.3}$$

The quality of the image should depend on a signal-to-noise ratio. We calculate the 'signal' as follows. The intensity received at the target T (at range l) lying in the common volume is

$$\left(P/\varepsilon^2 l^2\right)e^{-cl}g_1(\varepsilon) \tag{A.4}$$

where P is the power emitted by the floodlight. The symbols $g_1, g_2, g_3, \ldots$ will be used to denote dimensionless functions of their arguments; it is implied that all arguments are explicitly stated; there are no 'hidden' arguments. In addition, $g_1(\varepsilon)$ (like g_2 and g_3 below, but unlike g_4 onwards) has the property of being of order unity, that is, $g_1(\varepsilon)$ is bounded above and below by positive constants. In obtaining (A.4), we assume that the target is sufficiently near broadside and sufficiently far away that the distances of T from each of F, C and the plane FC are all equal, to a good approximation.

We assume that the surface of the target is parallel to the base plane and is made up of pieces, each of which scatters completely diffusely (i.e. the reflected power is spread over 2π steradians). The power received at the camera, which we take as the 'signal,' is therefore

$$\begin{aligned} \text{signal} &= \left[\left(P/\varepsilon^2 l^2\right)e^{-cl}g_1(\varepsilon)\right]\left[\left(T\rho S/l^2\right)e^{-cl}g_2(\ )\right] \\ &= \left(PTS\rho/\varepsilon^2 l^4\right)e^{-2cl}g_3(\varepsilon) \end{aligned} \tag{A.5}$$

Here T is the area of the target (taken to have its normal pointing approximately towards the floodlight and the camera), ρ is the average reflectivity of the target and S is the area of the camera aperture. $g_2(\ )$ is dimensionless with no arguments and could easily be calculated.

The detailed argument presented in the rest of this Section A.2 (which deals with the 'noise') makes use of the assumption that in turbid water, the attenuation is due almost entirely to scattering rather than absorption; thus $c_s \approx c$, where c_s and c are the scattering coefficient and attenuation coefficient respectively. Then the parameter c_s drops out of the problem in favour of c. After the calculation was complete it was pointed out that, as in Section 2, this assumption does not hold and that instead $c_s/c \approx 0.7$. However, it is not difficult to see the consequences of this change: the consequences are given in the footnote below Equation (5.1).

We now deal with the 'noise.' In turbid water the noise arises from the scattering of light from the floodlight, by scatterers in the common volume, directly into the camera. The bulk of the contributing scatterers lie at ranges such that

$$(\text{range of scatterer}) - (\text{range of K}) \sim 1/c$$

where K is shown in Figure 1. The left-hand side will be called the relative range. The result given by the display equation comes about for two reasons. First, scatterers closer to K (that is, at a relative range $\ll 1/c$) are relatively small in number. Second, while scatterers at relative ranges $r \gg 1/c$ are numerous, the intensity of the scattered light

received from them is very weak, because that intensity contains a factor e^{-2cr}, which is *exponentially* small.

It is seen that the geometry governing the noise involves two intersecting cones and is complicated; as a further complication, the attenuation introduces an exponential factor. Nevertheless, we shall see that dimensional considerations enable the essential information to be extracted.

The 'noise' is the scattered power reaching the camera. Obviously this noise is proportional to the power P of the floodlight, and also to the camera aperture area S provided that $\sqrt{S}$ is small compared to y_0. Thus we may write

$$\text{noise} = PS\,h_1(\varepsilon, y_0, c) \qquad (A.6)$$

where h_1 is some function (allowed to have dimensions) of its arguments. This form must hold because ε, y_0 and c are the *only* remaining parameters of the problem. But h_1 has the dimensions of length^{-2}. Dimensional analysis then allows us to proceed as follows. $y_0^2 h_1$ is dimensionless. It is therefore a function only of the independent dimensionless combinations that can be formed from the three arguments ε, y_0 and c. Thus (A.6) may be rewritten as

$$\text{noise} = (PS/y_0^2)\,g_4(\varepsilon, cy_0) \qquad (A.7)$$

where y_0^{-2} could equally well have been replaced by c^2. Note that g_4 contains only dimensionless arguments.

From (A.5) and (A.7), the signal-to-noise ratio is

$$\begin{aligned}
\text{SNR} &= (T\rho y_0^2/\varepsilon^2 l^4)e^{-2cl}g_5(\varepsilon, cy_0) \\
&= (T\rho/l^2)g_6(\varepsilon, y_0/l, cl)
\end{aligned} \qquad (A.8)$$

We now show how appropriate scaling simplifies the situation. When l is changed from the unscaled to a scaled value (or to several 'scaled' values), we have the option of changing ε and y_0, the characteristics of the video system, according to some law. We may also change T and ρ. Let us make these choices as follows. First, as argued in Section 3 (above Eqn 3.3), it seems necessary to make $T \propto l^2$ (geometric similarity). Then in (A.8) we have $T/l^2 = \text{constant}$ over the sequence of situations derived by scaling—a simplification. Again, the choice $\rho = \text{constant}$ seems eminently sensible. Now suppose that we also choose $\varepsilon = \text{constant}$ and $y_0/l = \text{constant}$, that is, we choose to scale the dimensions of the video arrangement (including, for example, l_0) in proportion to l. Then, *over the sequence of situations derived by scaling*, (A.8) gives

$$\text{SNR} = f_1(cl) \qquad (A.9)$$

Here f has been written in place of g as the unknown function, because: (i) f_1 has dimensions; but also (ii) unlike both g and h, f_1 contains 'hidden' dependences (on the 'constants' ε and y_0/l).

We now complete the argument. Constant quality throughout the sequence implies constant SNR; and from (A.9) this implies constant cl. Thus the scaling law (5.1) is obtained.

We note that the powerful method of dimensions, which was applied to the noise, might with profit be applied to the signal. This could lead to Equation (A.5)'s being generalised, for example to targets at shorter ranges l. (In the process, g_3 would presumably acquire further arguments.)

A.3. Quality of Acoustic Image

A.3.1 Instrumentation (Electronic) Noise

We consider the case where the instrumentation noise is dominant. First we calculate the 'signal.' The power falling on the target is proportional to

$$\left(T/l^2\right)e^{-\gamma l} \tag{A.10}$$

where T is the surface area of the target. (For simplicity, the target is assumed to be flat, directly facing the array, and located near broadside. We leave acoustic reflectivity out of the expression A.10, because the pattern of reflectivities over the target is taken to be the same in the unscaled as in the scaled target.) The power reaching the receiving array is then proportional to

$$\left[\left(TN\sigma/l^2\right)e^{-\gamma l}\right]\left[\left(1/l^2\right)e^{-\gamma l}\right]=\left(TN\sigma/l^4\right)e^{-2\gamma l} \tag{A.11}$$

where N is the number of elements and σ is the area of an element. The 'signal,' or intensity of the image at the target location, is proportional to this.

We now turn to the 'noise.' Let n be the instrumentation noise in the image intensity. The image quality is assumed to be determined by the signal-to-noise ratio. For equal quality in the unscaled and scaled images, we therefore require

$$\left(T^+N^+\sigma^+/n^+l^{+4}\right)e^{-2\gamma^+l^+}=\left(TN\sigma/nl^4\right)e^{-2\gamma l} \tag{A.12}$$

As argued in Section 3 for the optical case, it again seems necessary in the acoustic case to scale target size so as to preserve angular size; thus

$$T\propto l^2,\quad\text{i.e.}\quad T^+/T=\left(l^+/l\right)^2 \tag{A.13}$$

Then Equation (A.12) simplifies to

$$\left(N^+\sigma^+/n^+l^{+2}\right)e^{-2\gamma^+l^+}=\left(N\sigma/nl^2\right)e^{-2\gamma l} \tag{A.14}$$

At this point we note that if we scaled *all* the dimensions of the receiving device in proportion to l (as in Section A.2), we would have $\sigma\propto l^2$ and a further simplification in (A.14) would result. However in the end it is unattractive to pursue this route, since scaling the linear size of the array faces two grave problems. The first problem is that, when a small-scale replica of the AMI device is built, we can use (A.14) only if we know how the instrumentation noise n varies with size. But n varies in a quite unpredictable way, since n will vary with the details of the electronics. The second problem is that, in practice, the expense and time involved in designing and constructing a scaled-down instrument means that an experiment based on such a scaling-down is quite impractical.

Actually the scaling $\sigma \propto l^2$ is unsatisfactory in any case, because the beamwidth of an element would change with l, since $\sqrt{\sigma}/\lambda$ would change. Such a change of beamwidth is not acceptable in a scaling experiment. Suppose that, despite the immense difficulties, the scaled-down instrument could be built *and* the noise n kept constant. Then the argument just given shows that we would need to choose, not $\sigma \propto l^2$, but $\sigma^+ = \sigma$. This possibility is briefly discussed in Section A.3.3.

We now pursue the only practical route left open, that of keeping the *same* instrument (constancy of linear size). We *assume* that the instrumentation noise n is independent both of the power of the transmitted signal and of the power of the received signals (at constant transmitter power); therefore

$$n^+ = n \tag{A.15}$$

Since N and σ remain constant (same instrument), Equation (A.14) yields

$$l^{+-2} e^{-2\gamma^+ l^+} = l^{-2} e^{-2\gamma l} \tag{A.16}$$

as the condition for constant quality. Alternative forms of (A.16) are given as Equation (5.3). The scaling just discussed, which is the preferred scaling when there is just instrumentation noise, is summarised in Table 5. Note that unfortunately, the assumption made in this paragraph does not normally conform with reality: see the main text.

A.3.2 Clutter

Regarding the distant sidelobes (clutter), it is known that the image 'energy' in the point spread function is spread along the surface of N ellipsoids, each one being associated with a particular element. Each ellipsoid has as its foci: (i) the transmitter, and (ii) the element concerned. This ellipsoid structure is shown for a simple case in Figure 2.
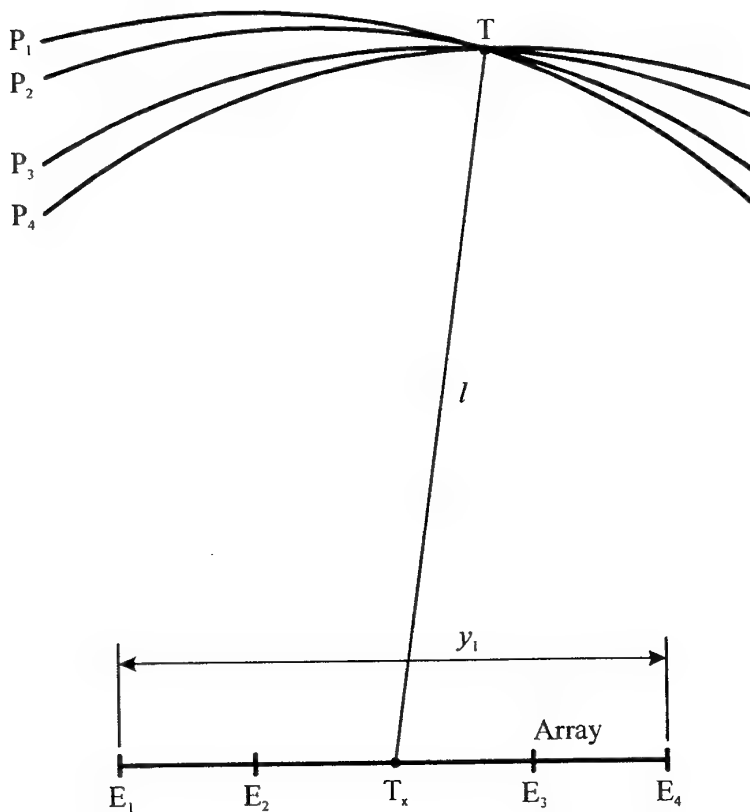
Figure 2: *Ellipsoid structure representing the sidelobes, drawn for a simple case. The linear array has four elements, E_1 to E_4, and an associated transmitter T_X. T is a point target. The image (point spread function) consists of ellipsoids. Each ellipsoid has two foci, one at the transmitter and one at an element; P_1 results from E_1, and so forth. Each ellipsoid is a surface of revolution about the line joining its foci. Each ellipsoid arises because it is the locus of the condition: go-and-return path = constant. Constructive interference of the ellipsoids at T yields the main peak.*

When scaling, to preserve the amount of clutter relative to the peak image intensity, it seems essential to preserve the structure of ellipsoids that results when the dimensions of the ellipsoids are expressed relative to the range l. To see that this is required, note, from Sections 3 and A.3.1, that the 'target' in the scaled system is a scaled model of the target in the unscaled system—scaled down in proportion to l. We require that, given the set of spots in the target that contribute to the clutter at a given point in the image, the corresponding set of spots in the model contributes after scaling. Hence it is necessary to preserve the ellipsoids.

However, for that ellipsoid structure to be preserved, it can be seen with the aid of Figure 2 that *the array must be scaled in proportion to l; i.e. $y_1 \propto l$*, where y_1 is a characteristic size of the receiving array. Essentially, this is because, when the whole of Figure 2 is scaled (similarity transformation), the ellipsoid property is preserved. The

relation $y_1 \propto l$ may also be written as $y_1 = Y_1/s$, where $s = l^+/l$ and Y_1 is the characteristic size of the actual AMI array.

The above argument for the scaling law $y_1 \propto l$, and for the more complete set of scaling laws (A.17) below, can be strengthened by considering the intensity pattern across the ellipsoids, but this strengthened argument will not be presented here.

We must also specify the scaling of N and σ. We first state the result, then give the reasoning. For the case where the only noise is clutter, the set of scaling laws for constancy of image quality appears to be:

$$y_1 \propto l, \qquad N = \text{constant}, \qquad \sigma = \text{constant} \qquad (A.17)$$

In (A.17), N is a key parameter in characterising clutter; for example, in the region, not far from the peak of the image, where all the 'ellipsoids' (each having an associated thickness) overlap strongly, the average intensity is $1/N$ times the intensity at the peak. Hence N must be kept constant for constant quality. Despite the scaling of y_1, σ must be kept constant, as shown in Section A.3.1. The scaling laws just derived—those appropriate when clutter noise is dominant—are summarised in Table 5. It is interesting to note that the attenuation constant γ does not enter the scaling laws (A.17); this is because an arbitrary variation of γ does not affect the size of the clutter *relative to* the main peak.

Of course, the scaling of the array in proportion to l, implied by (A.17), is quite impractical, as has been discussed in Section 5.2.1 and in Section A.3.1.

A.3.3 Combination of Instrumentation Noise and Clutter

Now consider the case where both noise sources are present. Suppose that the receiver is scaled according to (A.17), so that 'the demands' of clutter on scaling are met. Impractical though the scaling $y_1 \propto l$ is, let us see whether the demands of instrumentation noise (subject to the two assumptions above A.15) can simultaneously be met. The answer can be seen from Equation (A.14). Suppose we can and do build the scaled device so that $n^+ = n$ (Eqn A.15). Substituting this equality and the last two equalities of (A.17) into (A.14), we obtain, for this new situation, a scaling law identical to (A.16). We therefore expect that the combination of noise sources could (theoretically) be the subject of a scaled experiment. For the combination, the scaling laws for constant quality consist of the relations (A.17) and (A.15) together with (A.16). This result is better expressed as follows. The scaled devices should be built so that (A.17) and (A.15) hold. Then scaling for constant quality is achieved by (A.16). The above set of scaling laws is summarised in Table 5.

Unfortunately, as noted earlier, the feature $y_1 \propto l$ makes such scaling impractical.

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in the Context of Image Quality

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